NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

TECHNICAL NOTE

No. 1796

THEORETICAL ANALYSIS OF OSCILLATIONS OF A TOWED CABLE

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u	real part of root of equation (see formula (12))
U	velocity of propagation of wave along cable
v	imaginary part of root of equation (see formula (13))
V	airspeed
x	distance along cable
y	distance perpendicular to cable
α	angle of attack of cable

$$\gamma = \cos \alpha_0 + \frac{2 \sin^3 \alpha_0}{\sin 2\alpha_0}$$

$$\eta = \frac{KV_0^2}{2T}$$

λ	root of equation (u + iv)
μ	mass of cable per unit length
Λ	wave length of oscillation of cable
ρ	air density
ω	circular frequency of forced oscillation of cable

Subscript:

o	equilibrium value

ANALYSIS

When a disturbance is introduced at the end of a long cable which is in tension, the wave form of the disturbance remains unchanged as it travels down the cable provided that aerodynamic forces acting on the cable are neglected. A physical picture of the effect of aerodynamic forces on a cable in an air stream may be obtained from figure 1. In this figure a wave is shown at successive instants traveling down a cable in the same direction as the air velocity. This figure shows that if the airspeed is considerably greater than the speed of the wave the aerodynamic force on each element of the cable acts in the same direction as the transverse (in this case, vertical) motion of the element. The air force

instability has occurred has been found to depend on the type of airplane from which the body is lowered, the flap and power settings, and the point on the airplane from which the body is suspended. It has been thought that this violent motion was caused by the action of the turbulent wake of the airplane on the cable, inasmuch as the cable trails almost straight back from the airplane at high airplane speeds. The violence of the motions of the towed body has, however, been difficult to explain on this basis alone because the oscillation usually appears to be small near the airplane. The possibility was therefore investigated that oscillations of the cable originating near the airplane might be amplified by the action of air forces as they were propagated down the cable.

SYMBOLS

A undetermined function of x

a velocity of wave propagation along cable in vacuum $\left(\sqrt{T/\mu}\right)$

C constant

c abbreviation used in trigonometric derivation $\left(\alpha_0\right)$

C_D drag coefficient of cable when perpendicular to air stream

d abbreviation used in trigonometric derivation

$$\left(\frac{\partial y}{\partial x} + \frac{\cos \alpha_0}{V} \frac{\partial y}{\partial t}\right)$$

D diameter of cable

F force

e base of natural logarithms

g acceleration due to gravity

$i = \sqrt{-1}$

$K = C_D \dfrac{\rho}{2} D \sin 2\alpha_0$

P period of oscillation of cable

S equivalent flat-plate area

T tension in cable

t time

negative values of α. The neglect of the reversal in force at negative values of α will not lead to any difficulty inasmuch as the cable will be assumed to have an initial positive angle of attack so that the angle of attack will never become negative when the cable performs small oscillations.

The angle of attack of the cable element is

$$\alpha = \alpha_O - \frac{\partial y}{\partial x} - \frac{\cos \alpha_O}{V} \frac{\partial y}{\partial t}$$

In this expression, α_O may be a large angle but the remaining terms are assumed to be small. Hence, the expression for α has the form

$$\alpha = c - d$$

where d is a small quantity. The expression $\sin^2 \alpha$ is given by the trigonometric formula:

$$\sin^2 \alpha = \sin^2(c - d) = \frac{1}{2} - \frac{\cos 2c \cos 2d + \sin 2c \sin 2d}{2}$$

Since d is small, $\cos 2d = 1$ and $\sin 2d = 2d$
Hence,

$$\sin^2 \alpha = \frac{1}{2} - \frac{\cos 2c + 2d \sin 2c}{2}$$

$$= \sin^2 c - d \sin 2c$$

The values for c and d may be placed in this expression as follows:

$$\sin^2 \alpha = \sin^2 \alpha_O - \left(\frac{\partial y}{\partial x} + \frac{\cos \alpha_O}{V} \frac{\partial y}{\partial t}\right) \sin 2\alpha_O \tag{4}$$

The relative velocity of the cable element and the air is given by expression:

$$V = V_O - \frac{\partial y}{\partial t} \sin \alpha_O$$

If higher-order terms are neglected,

$$V^2 = V_O^2 - 2V_O \frac{\partial y}{\partial t} \sin \alpha_O \tag{5}$$

would therefore be expected to feed energy into the oscillation and thereby to increase the amplitude of the oscillation as it travels down the cable. The conclusion may be reached from similar considerations that if the airspeed is the same as the speed of propagation of the wave no aerodynamic effects will be present, because in this case there is no relative transverse motion between the air and the elements of the cable. If the airspeed is less than the speed of propagation of the wave, the air forces will have a slight damping effect.

A more detailed analysis is necessary to determine the rate of damping or amplification of the waves as a function of airspeed and also to find whether the aerodynamic forces affect the speed at which the wave travels down the cable. For this purpose the differential equation of motion of the cable will be set up. The method of analysis is similar to the treatment given in reference 2 of the propagation of transverse waves along a cable where the motion of elements of the cable is resisted by viscous damping forces.

The method of calculating the forces on an element dx of a cable undergoing a small transverse oscillation is illustrated in figure 2. In its equilibrium position, the cable lies along the x-axis and makes an angle α_o with air stream of velocity V_o. The forces acting normal to the cable are now determined. The transverse force on an element of the cable due to tension in the cable is

$$dF_{(tension)} = -T \frac{\partial y}{\partial x} + T \left(\frac{\partial y}{\partial x} + \frac{\partial}{\partial x} \frac{\partial y}{\partial x} dx \right) = T \frac{\partial^2 y}{\partial x^2} dx \tag{1}$$

The gravitational force normal to an element of the cable, if the air stream is assumed to be horizontal, is

$$dF_{(gravity)} = -\mu g \, dx \cos \alpha_o \tag{2}$$

The aerodynamic force is assumed to act normal to the cable element and is given by the formula:

$$dF_{(aerodynamic)} = C_D \frac{\rho}{2} V^2 D \, dx \sin^2 \alpha \tag{3}$$

The variation of force with $\sin^2 \alpha$ is in accordance with the concept that the force on the cable is due to the component of relative velocity normal to the cable. Measurements of the air forces on cables have shown good agreement with this law (reference 3). This formula holds only in the positive range of angles of attack. The force should vary as $-\sin^2 \alpha$ for

may be balanced partly by gravity or partly by a gradual curvature of t cable, as represented by a constant portion of the first term. This curvature will cause a variation of angle of attack along the cable. A the tension may vary along the cable. It will be assume., however, tha cable in equilibrium is straight and that the angle of a tack α_o and tension are constant. Calculations of cable shape given in reference 1 that a cable towing a body behind an airplane is fairly straight over a large part of its length. In any case, the analysis of he cable motio over a region of the cable should be approximately corre t if the angle of attack is taken as that existing over the portion of he cable under consideration and if the radius of curvature is large cc pared to the wave length of the disturbances that are being considere.

To further simplify the notation let the trigonomet ic term

$$\cos \alpha_o + \frac{2 \sin^3\alpha_o}{\sin 2\alpha_o} \quad \text{equal} \quad \gamma. \quad \text{Also let}$$

$$a^2 = \frac{T}{\mu}$$

and

$$\eta = \frac{KV_o^2}{2T}$$

After the steady-state terms are omitted and the remaini g terms rearranged, equation (7) becomes

$$\frac{\partial^2 y}{\partial x^2} - 2\eta\frac{\partial y}{\partial x} - \frac{2\eta\gamma}{V_o} \frac{\partial y}{\partial t} = \frac{1}{a^2} \frac{\partial^2 y}{\partial t^2} \qquad (8)$$

Equation (8) is the differential equation governing smal transverse deviations of the cable from its equilibrium position. his equation differs from the equation for a cable with no aerodynami forces by the addition of the second and third terms, which depend on erodynamic eff The equation without these two terms is the well-known w ve equation. solution of the wave equation shows that waves of arbitr ry shape trave along the cable in either direction with the velocity a and that the and amplitude of the waves remain unchanged as they trav l. (See, fo1 example, reference 4.)

In order to study the propagation of waves along a able in an air stream, it will be assumed that one point on the cable i forced to oscillate sinusoidally. The cable is assumed to extend rom this poir.t

The aerodynamic force on the cable element, obtained by substituting the value for $\sin^2\alpha$ from formula (4) and the value for V^2 from formula (5) in formula (3), is as follows:

dF(aerodynamic)

$$= C_D \frac{\rho}{2} D\left(V_o^2 - 2V_o \frac{\partial y}{\partial t} \sin \alpha_o\right)\left[\sin^2\alpha_o - \left(\frac{\partial y}{\partial x} + \frac{\cos \alpha_o}{V_o - \frac{\partial y}{\partial t} \sin \alpha_o} \frac{\partial y}{\partial t}\right)\sin 2\alpha_o\right] dx$$

If higher-order terms are neglected,

$$dF\text{(aerodynamic)} = C_D \frac{\rho}{2} DV_o^2 \, dx \, \sin^2\alpha_o - \frac{\partial y}{\partial x} C_D \frac{\rho}{2} DV_o^2 \, dx \, \sin 2\alpha_o$$

$$- \frac{\partial y}{\partial t} C_D \frac{\rho}{2} DV_o \left(\sin 2\alpha_o \cos \alpha_o + 2 \sin^3\alpha_o\right) dx \qquad (6)$$

For brevity, let

$$K = C_D \frac{\rho}{2} D \sin 2\alpha_o$$

The equation of motion of the cable element, obtained by equating the force due to tension, equation (1), the gravitational force, equation (2), and the aerodynamic force, equation (6), to the inertia force on the element, is

$$T \frac{\partial^2 y}{\partial x^2} dx - \mu g \, dx \cos \alpha_o + C_D \frac{\rho}{2} DV_o^2 \, dx \, \sin^2\alpha_o - \frac{\partial y}{\partial x} KV_o^2 dx$$

$$- \frac{\partial y}{\partial t} KV_o \left(\cos \alpha_o + \frac{2 \sin^3\alpha_o}{\sin 2\alpha_o}\right) dx = \mu \frac{\partial^2 y}{\partial t^2} dx \qquad (7)$$

The term dx may be canceled. The resulting equation is one of the differential equations governing the shape and motion of the cable. Another equation, obtained from the equilibrium of forces acting along the cable, would be required to obtain a solution for the cable shape. This additional equation will not be used in the present analysis, however, because this analysis is concerned only with the motion of the cable, which involves deviations from its equilibrium condition. For the same reason, the steady-state terms will be dropped from equation (7). When the cable is in equilibrium, the steady aerodynamic force, given by the third term,

Let $\lambda = u + iv$. The following expressions are found for u and v:

$$u = \eta\left(1 + \frac{\gamma\omega}{V_o v}\right)$$

$$v = \pm\sqrt{\frac{1}{2}\left(\frac{\omega^2}{a^2} - \eta^2\right) \pm \sqrt{\left[\frac{1}{2}\left(\frac{\omega^2}{a^2} - \eta^2\right)\right]^2 + \gamma^2\frac{v^2\eta^2}{V_o^2}}}$$

By combining the solutions for the dependence of the cable position on values of t (equation (9)) and x (equation (11)), the motion of the cable is represented by the expression

$$y = Ce^{(u+iv)x+i\omega t}$$

Or, if the real part of this expression is taken as the actual motion of the cable,

$$y = Ce^{ux}\cos(\omega t + vx)$$

DISCUSSION OF RESULTS

The solution for the motion of the cable, equation (14), indicates that trains of waves are traveling up and down the cable from the origin where the sinusoidal disturbance is applied. The waves traveling down correspond to a negative value of the term v, and those traveling up correspond to a positive value, as explained in reference 4.

The value of u, which determines the rate of increase or decrease amplitude of the waves with distance from the origin, is different for the two sets of waves, because of the occurrence of the term v in the expression for u (formula (12)). When v is positive, corresponding to waves traveling upwind, both terms in the formula for u are positive. The amplitude of the oscillations, which is proportional to e^{ux}, always decreases with increasing distance upwind from the origin because this direction corresponds to negative values of x.

When v is negative, corresponding to waves traveling downwind terms in the formula for u have opposite signs. For small values of airspeed, u will be negative, and the oscillation will therefore decrease with increasing distance downwind from the origin, because this direction

to infinity in either direction. A solution will be obtained for the resulting steady-state oscillation of the cable.

In a steady-state forced oscillation, all elements of the cable oscillate with the forcing frequency. For purposes of analysis, this oscillation is represented in the exponential form:

$$y = Ae^{i\omega t} \tag{9}$$

where A is a function of x. This expression gives the time dependence of the motion of the elements of the cable, hence

$$\frac{\partial y}{\partial t} = Ai\omega e^{i\omega t} = i\omega y$$

and

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2 e^{i\omega t} = -\omega^2 y$$

If these expressions are substituted in equation (8), the equation becomes

$$\frac{\partial^2 y}{\partial x^2} - 2\eta \frac{\partial y}{\partial x} - \left(\frac{2\eta\gamma}{V_o} i\omega - \frac{\omega^2}{a^2}\right)y = 0 \tag{10}$$

The solution of this equation gives the position dependence of the motion of the cable elements. This solution has the form:

$$y = Ce^{\lambda x} \tag{11}$$

If this value is substituted in equation (10), the values of λ are obtained as roots of the equation

$$\lambda^2 - 2\eta\lambda - \left(\frac{2\eta\gamma}{V_o} i\omega - \frac{\omega^2}{a^2}\right) = 0$$

whence

$$\lambda = \eta \pm \sqrt{\eta^2 + \frac{2\eta\gamma}{V_o} i\omega - \frac{\omega^2}{a^2}}$$

This expression must be separated into its real and imaginary parts.

the velocity of wave propagation, waves traveling downwind will be da
When the airspeed is greater than the velocity of wave propagation, w
traveling downwind will increase in amplitude.

In the practical problem of an airplane towing a body by means of
cable, disturbances introduced at the airplane travel downwind along
cable until they reach the body. A part of these waves is then refle
upwind, the nature of the reflected wave depending on the amount of r
provided by the body. The solution shows that the wave traveling up
well damped compared to the waves traveling downwind. The amplitude
waves near the airplane, and in practice, over the grea er part of th
cable, may therefore be studied by considering only the waves moving
wind and neglecting the reflected waves or the nature o the restrain
far end of the cable. The value of u is proportional to η (formu
But

$$\eta = \frac{KV_0^2}{2T} = \frac{C_D \frac{\rho}{2} DV_0^2 \sin 2\alpha_0}{2T}$$

For small angles of attack, $\sin 2\alpha_0$ is proportional to α_0. Hence f
small angles of attack, the value of u, which determine s the rate of
amplification of the oscillations, is proportional to . A towed c
will ordinarily be pitched with respect to the air stre n but not yaw
Any oscillations that occur would therefore be expected to take place
vertical plane. In practice, the unstable motion of to ed airspeed h
at high speeds has been observed to involve longitudinal motions pre-
dominantly.

The predicted lack of amplification of oscillation at zero angl
attack results from the assumption of infinitesimal osc llations and
the assumption that the aerodynamic force varies as si $^2\alpha$. Thus, the
lift-curve slope for the cable is assumed to be zero at zero angle of
It may be expected that oscillations of finite amplitude will be ampl f
even at zero angle of attack because such oscillations will result in t
development of aerodynamic forces on the cable elements.

Typical effects of airspeed on waves traveling alon a towed cab
have been calculated. The characteristics of the cable vere taken as t
of a cable used in conjunction with towed airspeed head These charac
teristics are as follows:

D, inch . . . .
μ, slug per foot . . . . 0.(
T, pounds . . . .
C_D . . . .
ρ, slug per cubic foot . . . . 0.(

corresponds to positive values of x. For high values of airspeed, however,
u will be positive, and the oscillation will increase with increasing dis-
tance downwind from the origin.

The wave length of the disturbance propagated along the cable is

$$\Lambda = - \frac{2\pi}{v}$$

The period is

$$P = \frac{2\pi}{\omega}$$

Hence, the velocity of propagation of the waves is

$$U = \frac{\Lambda}{P} = - \frac{\omega}{v}$$

If U is substituted for $-\frac{\omega}{v}$ in the formula for u (formula (12)), the
formula becomes

$$u = \eta\left(1 - \frac{\gamma U}{V_o}\right)$$

The value of γ is plotted as a function of α_o in figure 3. For
small values of α_o the value of γ is very close to 1.0. In this case
the value of u is approximately

$$u = \eta\left(1 - \frac{U}{V_o}\right)$$

The value of η is proportional to V_o^2. The dependence of the value of u
on the velocity is therefore given by an expression of the form:

$$u \propto V_o^2 - UV_o$$

It may be seen that when $V_o = U$ (that is, when the airspeed is the same
as the velocity of wave propagation) the value of u is zero. Under these
conditions a disturbance travels downwind along the cable without change in
amplitude. The solution therefore agrees with the physical concept that
for small angles of attack the aerodynamic forces will have no effect when
the wave moves at the same speed as the air. When the airspeed is less than

cable elements to have a component of motion parallel to the wind direc
Damping forces are produced by this motion which cause t e speed for
instability to increase again.

An attempt has been made to determine whether the p edictions of t
theory are in reasonable agreement with the observed cha acteristics cf
towed airspeed heads. It was mentioned previously that he maximum spe
at which a towed airspeed head of the type described has been successfu
used depended on the type of airplane from which it is s ispended, on th
flap and power settings, and on the location from which t is lowered.
a large airplane in the flap-down condition, which would be expected to
produce a large disturbance in the wake, the airspeed he d has become
unsteady at speeds as low as 135 miles per hour. In thi case the moti
was both lateral and vertical. In the case of an airpl e in the clear
condition in which the airspeed head was lowered from a osition near
trailing edge of the wing, the head became unstable at 1 5 miles per hc
The maximum speed at which such an airspeed head has eve been known to
remain stable is 275 miles per hour. When the instabili y occurs at hi
speed, it involves vertical motions predominantly.

Consideration of these facts leads to the belief th it when the
instability occurs at low speeds it may be due largely t the direct
effect of disturbance in the wake; whereas when it occu s at high speed
it is probably the result of a small oscillation caused y a disturbanc
acting on the cable near the airplane and amplified by t e air forces
acting on the cable. Inasmuch as the amplitude of the forcing motion i
unknown in any particular case, a definite correlation l tween the the c
and the observed characteristics of the airspeed head i not possible.
The agreement of the theory with the observations may be shown to be
reasonable, however, on the basis of the very rapid incr ase in amplifi
tion if the speed is increased beyond the values which l ave been reacle
in practice.

The curves shown in figure 4 are for a cable which is assumed to b
straight. Usually, the cable will be curved. In order to estimate the
rate of amplification of a wave traveling down a curved cable, it would
be necessary to divide the cable into a series of segmens each of whic
might be considered straight and to estimate the amplif ation taking p
in each segment. This calculation may be made with the aid of a series
of curves such as those shown in figure 5, which presens the cable len
required for a disturbance to double in amplitude as a function of air
for various angles of attack of the cable. These curve were compute
the same cable characteristics as those assumed previou ly.

Calculations have been made to indicate the charac eristics of the
assumed cable and airspeed head at an airspeed of 270 m les per hour be
this speed is close to the maximum speed at which this airspeed head as
been successfully used. The equilibrium shape of such cable 200 feet

In estimating the speed of propagation of the waves from the value of v

(formula (13)) it was found that the term $\dfrac{\gamma^2\omega^2\eta^2}{V_o{}^2}$ was very small compared

with other terms in the expression. The value of v then is almost
exactly

$$v = \pm \sqrt{\frac{\omega^2}{a^2} - \eta^2}$$

Furthermore, for values of ω corresponding to frequencies greater than

3 cycles per second, the value of η^2 is small compared with $\dfrac{\omega^2}{a^2}$. For the

assumed cable, then, and for frequencies of oscillation greater than 3 cycles
per second, the value of v is approximately

$$v = \pm \frac{\omega}{a}$$

The velocity of the waves is therefore

$$U = -\frac{\omega}{v} = \pm a$$

Hence the velocity of propagation of the waves is nearly the same for the
towed cable in the air stream as it would be for the cable in a vacuum.
For the cable under consideration, $a = 135$ feet per second or 92 miles per
hour.

At small angles of attack, the waves traveling down the cable will
be amplified at airspeeds greater than 92 miles per hour. In order to
show the extent of the amplification at various airspeeds and angles of
attack, the amplitude of oscillation at a point on the cable 200 feet from
the point of attachment to the airplane has been calculated for a unit
amplitude of disturbance at the point of attachment. These results are
shown in figure 4.

These curves show that for each angle of attack of the cable, the
amplitude of the oscillation increases very rapidly as the speed is
increased beyond a certain value. This rather abrupt onset of violent
motion above a certain speed is apparently in accordance with the observed
behavior of towed airspeed heads. The speed at which the oscillation is
rapidly amplified is shown in figure 4 to decrease as the angle of attack
of the cable is increased up to about 30°, but this speed increases slightly
at 45°. The initial reduction in the speed for instability is caused by
the increase of lift-curve slope of the cable with increase in angle of
attack. At large angles of attack, however, the oscillation causes the

cable. This method would reduce the cable angle and incr ase the speed propagation of waves. It would also reduce the response f the body to given cable disturbance. A possible disadvantage of this method would b that of allowing the cable to be more influenced by the w ke of the airp This problem might be avoided, however, by supportirg the cable on a st below the fuselage so that it would remain clear of the w ke at all tim

The preceding discussion has been concerned mainly w th towed airs heads. The applications of the theory to other towed dev ces, such as gliders and targets, is now discussed. A towed airspeed ead is ordina a small, heavy, streamlined body whioh has small drag in omparison wit its weight. As a result, this type of towed body introdu es a tension force at the far end of the cable which is fairly indepen ent of airspe On the other hand, gliders and targets have relatively la ge drag. Thes bodies cause a tension force in the cable which increases as the square the airspeed. The speed of propagation of waves alcng th cable, giver

by $\sqrt{T/\mu}$, would therefore increase directly with the airs eed. It has been shown that the cable oscillations will not be amplif ed if the spe of the waves is greater than the airspeed. If the drag o the towed bol is sufficiently large, therefore, the cable oscillations ill be stable any airspeed. The equivalent flat-plate area of the body required to m the velocity of the waves equal to the airspeed is

$$S = 1.56 \frac{\mu}{\rho}$$

In practice, a smaller drag might be sufficient, inasmuch as the cable oscillations are not amplified rapidly until the airspeed is considerab greater than the speed of wave propagation.

No cases are known in which cable oscillations have roved undesir il in the towing of gliders or targets. In these applicatic s, the drag o the towed bodies has presumably been great enough to prov de stability cable oscillations.

CONCLUDING REMARKS

It has been found from a theoretical study of the os illations of towed cable, that oscillations traveling downwind along t e cable are a by aerodynamic forces when the airspeed is greater than t e speed of pr pagation of waves along the cable. The oscillations are lightly dampe l when the airspeed is less than the speed of wave propagat on. Waves traveling upwind along the cable are always damped, and t e damping inc with increasing airspeed.

long when it is towing a 15-pound airspeed head has been estimated by means of a step-by-step solution. This analysis showed that at an airspeed of 270 miles per hour the cable is essentially straight over all except the final 30 feet of its length. The straight portion of the cable trails at an angle such that the cable weight is supported by the aerodynamic force on the cable. This angle varies inversely as the speed and is 5° at 270 miles per hour. The final 30 feet of the cable hooks down slightly due to the weight of the airspeed head. The tension in the cable increases from 16 pounds at the lower end to about 45 pounds at the airplane. The amplification of waves traveling down the cable may therefore be estimated quite well simply from the curve of figure 5 for a tension of 30 pounds and an angle of attack of 5°. This curve indicates that the amplitude would double each 52.3 feet, so that at a point 170 feet from the airplane, the amplitude would be about 9.5 times the forcing amplitude. This amplitude would probably be doubled again in the more steeply inclined portion at the end of the cable, though accurate estimations of the motion near the end have not been attempted, because reflected waves probably influence the motion in this region. It may be concluded, however, that an amplification of roughly 20 times the forcing amplitude would occur at this speed. The rapid increase in amplitude with further increase in speed makes it appear reasonable that 270 miles per hour is close to the maximum speed at which this type of airspeed head may be used.

Methods of increasing the speed at which instability of a towed body occurs may be examined on the basis of the theory. One method which has been suggested is to increase the weight of the towed body. This method has been proposed with the object of causing the cable to hang at a steeper angle so that it will be less influenced by the wake of the airplane. It has been shown, however, that very large increases in weight over that ordinarily used for airspeed heads would be required to influence appreciably the cable angle at the airplane, inasmuch as the major portion of the cable is nearly straight and hangs at an angle determined by its own weight rather than by the weight of the suspended body. Also the increased weight would increase the length of cable inclined steeply to the air stream, thereby increasing the amplification of waves. On the other hand, increasing the weight of the body would increase the cable tension, which would have the beneficial effect of increasing the speed of propagation of waves along the cable which is equal to $\sqrt{T/\mu}$. Finally, a heavier body would respond less violently to cable oscillations, so that larger cable motion might be possible without being objectionable. Despite these advantages, it does not appear that increasing the weight of the towed airspeed head within practical limits would greatly increase the speed at which it would remain steady in view of the rapid increase in cable instability with increasing speed.

A more promising means of increasing the speed at which violent motions of airspeed heads occur appears to be that of reducing the weight of the
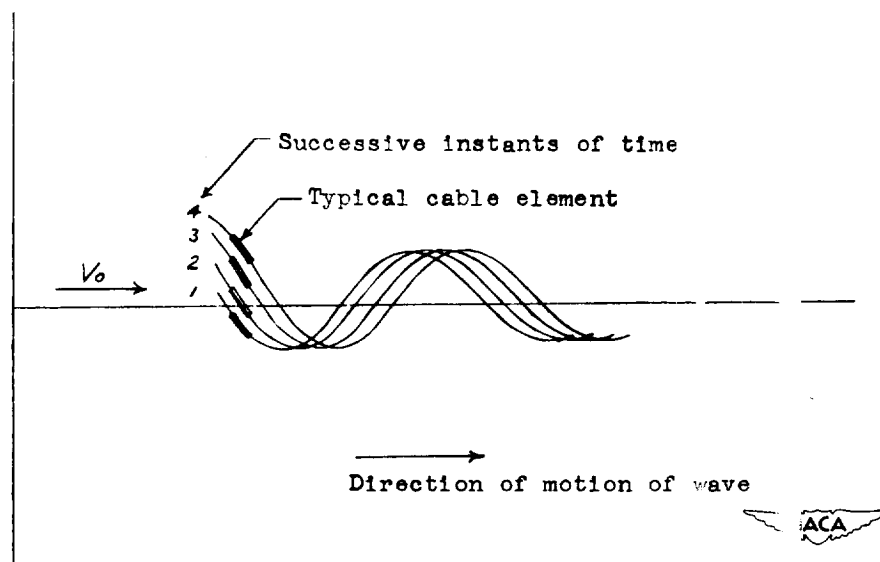
NACA TN No. 1796

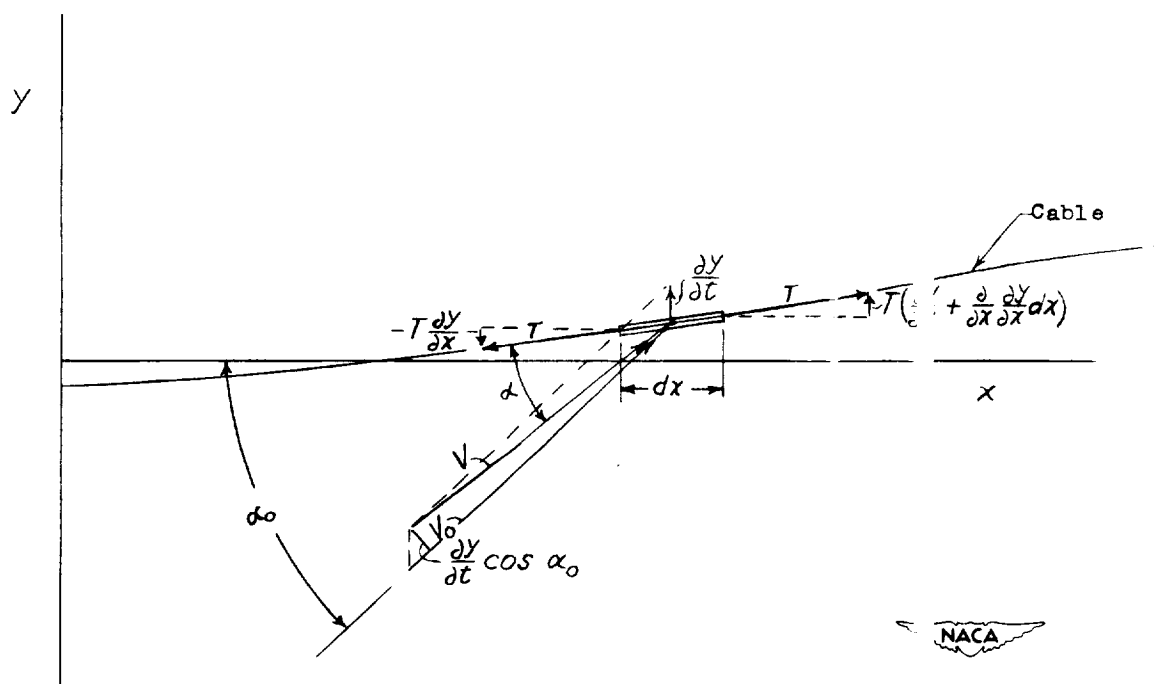Figure 1.- Motion of cable element as wave travels d wn cable.

Figure 2.- Forces acting on an element of the cable.

This theory provides a possible explanation for the violent motions of towed airspeed heads which appear above a certain speed. These oscillations are attributed to cable oscillations which originate near the airplane and are amplified by aerodynamic forces as they travel down the cable.

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    National Advisory Committee for Aeronautics
        Langley Air Force Base, Va., November 15, 1948

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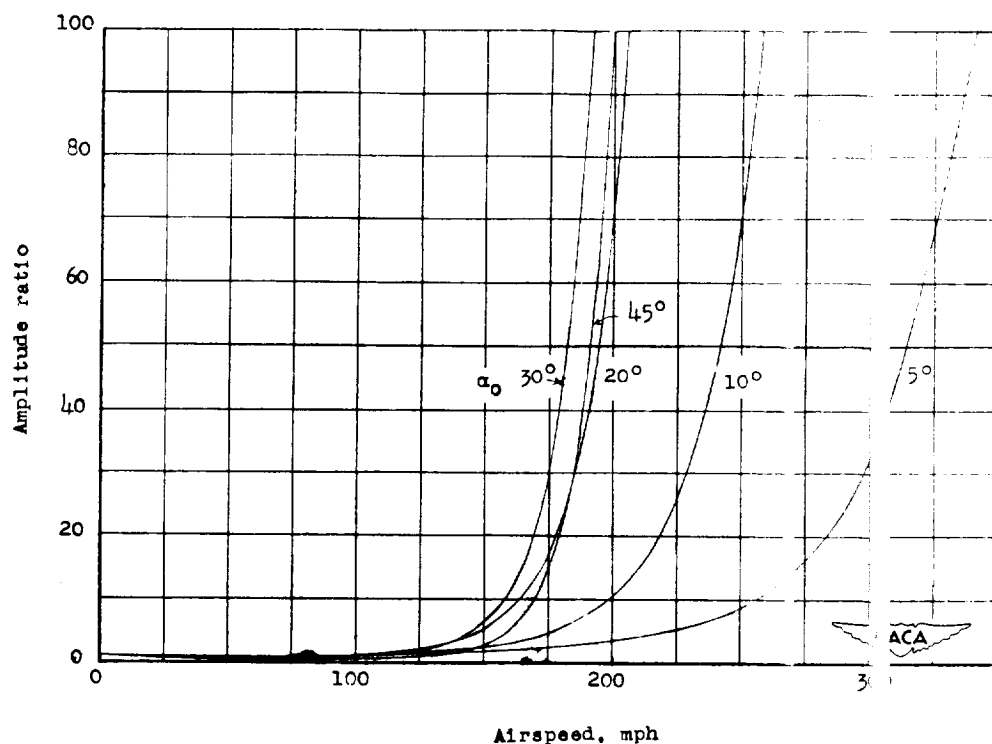
Figure 4.- Ratio of amplitude of oscillation at a point on cable 2) feet from airplane to forcing amplitude at airplane as a function of airs ied. Cable assumed straight.
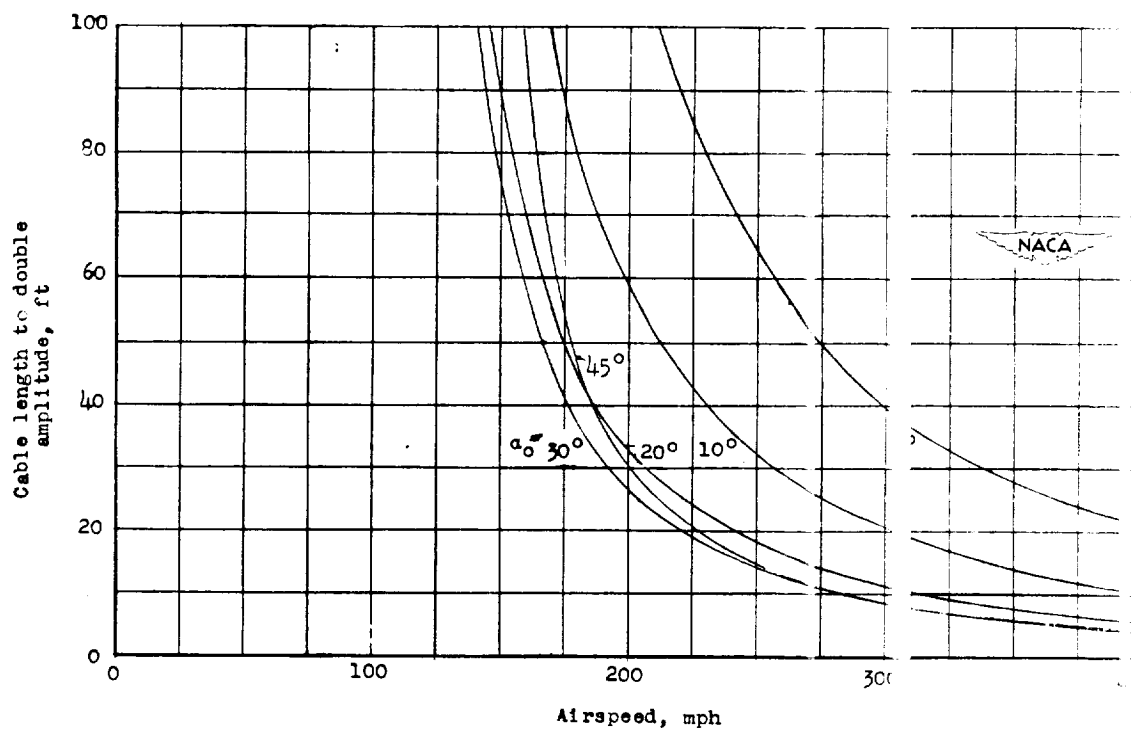

Figure 5.- Cable length in which oscillation doubl s in & plitude as a function of airspeed.
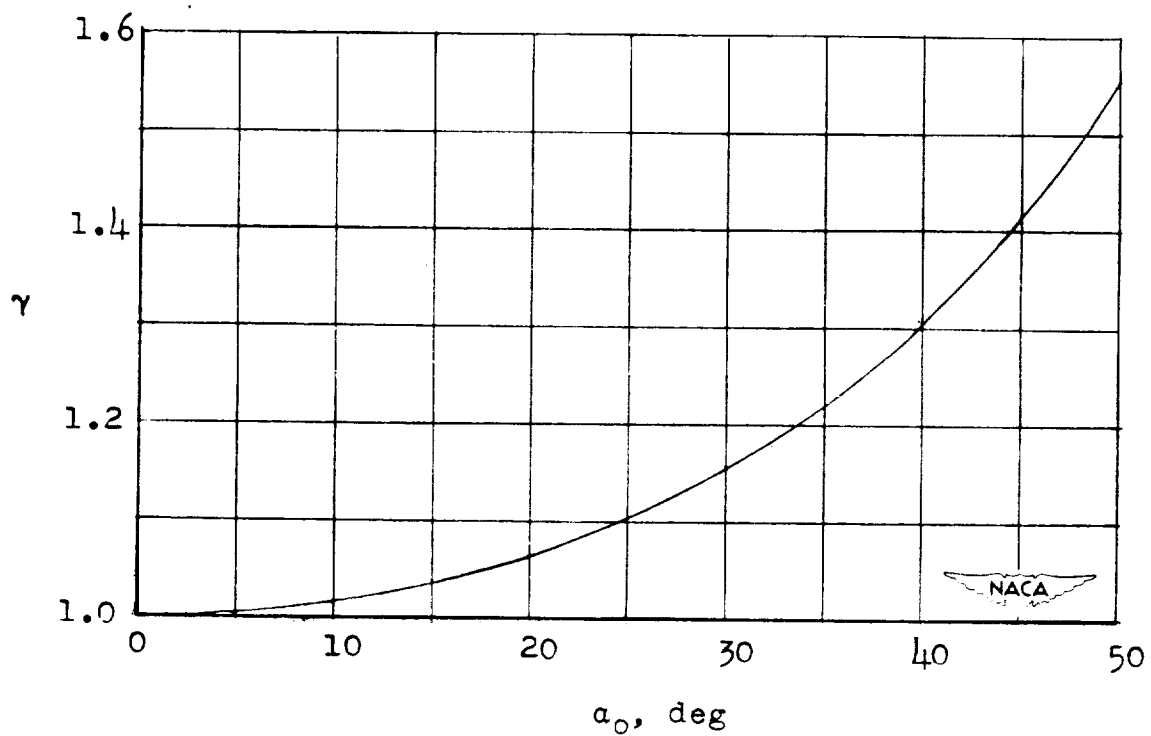
Figure 3.- Graph showing γ as a function of a_o.